

# Convergence Study of Biogeography Based Optimization

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**Abstract** - Biogeography based optimization BBO is a progressive algorithm. It is induced by Biogeography. BBO is more powerful algorithm among the biology based optimization methods. In this paper examines the convergence of BBO algorithm on some fitness functions. BBO algorithm handles the best solution from one off spring to the next converges to the universal optimum. The convergence rate evaluate of BBO algorithm by simulation for some fitness function. A set of 12 standard benchmark function performance of convergence is studied by BBO algorithm.

**Keywords:** BBO Algorithm, Migration, Mutation, Emigration

## I. INTRODUCTION

BBO relates a new type of evolutionary algorithm. A mathematical model of biogeography depicts the immigration and emigration of biological organism. It was first adduced in 2008[3]. BBO replicas after immigration and emigration of biological organisms between odor. It applies to the fitness of each organism result to obtain its immigration and emigration rate. The immigration rate obtains then a biological organism solution is to change its decision variables. It has showed better performance on many unconstrained and constrained benchmark functions. An optimization problem is a real world optimization problem. Some mathematical benchmark functions have been used to find Evolutionary Algorithm convergence. Mathematical modeling is based on biogeography based optimization, how the breed transfer from one place to another place. How new breeds grow and how destructed the breeds. There are two types of migration in BBO, like that emigration and immigration both are influenced by many factors e.g., distance of isle to the nearby neighbor, dimension of the isle, Habitat suitability index (HSI) etc. HSI depends on many elements like that vegetation, atmosphere, rainfall etc. These elements support the implicitness of breeds in a (habitat) provenance. Provenances are favorable for the living place of biological breeds. They will be advanced HSI [1]. Many number of breeds will be engaged by high HSI to Habitat.

In Breeds distribution, high HSI habitats are more consistent than low HSI habitats. There are so many opportunities on high HSI Isle for emigrating to near habitats. Biogeography is type of dispensation of breeds and it is similar to ordinary problem solutions. Suppose some relevant issues and their solutions are presented like that in economics, business, sports, medical science etc. A perfect rectification is similar to an isle with a high HSI and an imperfect rectification

describes an isle with a low HSI. Many researchers have been evaluated and examined the performance of BBO on several benchmark functions. Guo and Yu [2] Considered optimization algorithm converge to the universal optimum, When the population limit tends to infinity then candidate solution exist for at least one value, which provide the global solution using various mathematical tools of the optimization problem. Simon [3] considered the natural biogeography and its related mathematical tools, how to solve optimization problem.

Ma and Simon [4] BBO is a new approach of EA for getting best performance on various unconstrained and constrained benchmark functions have been studied. Ma [5] demonstrated the migration models in BBO and explores the execution through different benchmark functions. Simon et.al [6] derived the proportion of each individual in the population for a given optimization problem using theory rather than simulation dynamic system model for biogeography-based optimization (BBO). Ilhem *et al.*, [7] Evaluated the result for given objective function for the constrained optimization i.e. inequality and equality problems by BBO. Ma *et al.*, [8] described BBO for multi objective optimization problem wherein proposed algorithm is used to non dominated sorting approach to improve the convergence efficiency. Hordri [9] investigated the performance of BBO, GA and PSO for convergence. Feng *et al.*, [10] proposed modified Biogeography-Based optimization with Local Search Mechanism for migration operator in BBO, through it more information can be extracted from other habitats. Guo *et al.*, [11] explained the effect of migration rates on BBO; these are useful for designing of migration model.

Ma *et al.*, [12] Analyzed the Biogeography-Based optimization for Binary Problems provide the best candidate in the population from one generation to the next which converges to the global optimum solution. Golafshani [13] explored the impact of the Biogeography Based Programming for several benchmark functions to solve the problems. Weian *et al.*, [14] Investigated migration models for Multi-Objective Problems (MOPs) using BBO. Ma and Simon [15] considered a BBO evolutionary algorithm papers since last 10 years wherein they summarized and organized the literature. Khademi *et al.*, [16] considering the significant and expanding research of BBO and its applications in different domain.

We organized the paper as follows. In section II, described the biogeography-based optimization (BBO) algorithm. Convergence study of biogeography-based optimization (BBO) algorithm is proposed, in section, III. Finally conclusion is drawn in section IV.

## II. BIOGEOGRAPHY BASED OPTIMIZATION (BBO)

As observed the Biogeography based optimization algorithm which is introduced by Dan Simon. It is a new appearing population based algorithm. In BBO algorithm, feasible result depends on the habitat and their characteristics. Their characteristics described with merits are called merit/suitability index variables (SIVs). In BBO, there are some common features with other biological based algorithms, like PSO, GAs and BBO. If each solution is desirable then it is known as habitat suitability index variables (SIVs).

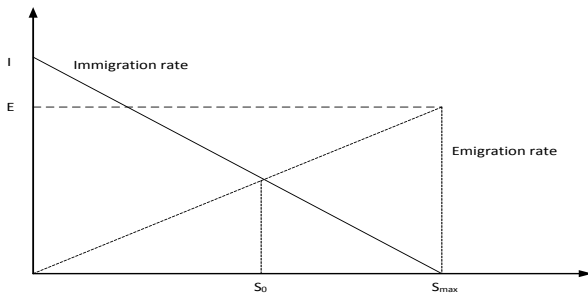


Fig. 1 Migration

### A. Migration

Migration is random operator. It is studied to enhance the candidate solution  $f_i$ . For every characteristics of a given candidate solution  $f_i$ , the candidate solution's immigration rate  $\lambda_k$  is used to randomly decide whether to immigrate or not. If the possibility immigration is there, then the emigration candidate solution  $f_j$  is randomly chosen based on the emigration rate  $\mu_j$ , it is given as

$$f_i(r) \leftarrow f_j(r) \quad \dots\dots (1)$$

where  $r$  is a symbol of candidate solution. In BBO, every candidate solution  $f_i$  has its own immigration rate  $\lambda_k$  and emigration rate  $\mu_k$ . A valid result has relatively high  $\mu$  and low  $\lambda$ , however the converse is true for a weak candidate solution. Immigration rate  $\lambda_k$  and emigration rate  $\mu_k$  of the candidate solution can be achieved by using (1)

$$\lambda_k = 1 - \text{fitness}(f_i)$$

$$\mu_k = \text{fitness}(f_i)$$

where fitness indicates candidate solution of fitness value. It's range is [0,1] The probabilities of immigrating to  $f_i$  and of emigrating from  $f_i$  are obtained as

$$\Pr(\text{immigration to } f_i) = \lambda_k$$

$$\Pr(\text{emigration from } f_i) = \frac{\mu_k}{\sum_{j=1}^N \mu_j}$$

where  $N$  is population size.

### B. Mutation

Mutation is random operator that randomly improves a candidate solution characteristic; the objective of mutation is to enhance diversity among the population. Each population candidate is related randomly, which denotes the presumption that it was required to exist as a result to the given problem. Very high HSI results and very low HSI results are not possible to equal improbable. Medium HSI results are expected to be probable. In a case result  $R$  has a low probability  $P_R$ , and then astoundingly it exists as a result. It seems to mutate to some other result. Other than, a solution with a high probability has less chance to mutate to different results. It is denoted as mutation rate  $m$ .

$$m(R) = m_{\max} \left( \frac{1 - P_R}{P_{\max}} \right)$$

Where  $M_{\max}$  is a user defined parameter. Mutation approach makes low HSI solutions likely to mutate which gives them a chance of improving. It also provides high HSI solutions likely to mutate which gives them a chance of improving even more than they already have.

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Initialize a population of N Candidate Solution {X_k}
While not(termination criterion)
For each X_k set emigration probability  $\mu_k \propto$  fitness of  $X_k$  with  $\mu_k \in [0, 1]$ 
For each X_k set immigration probability  $\lambda_k = 1 - \mu_k$ 
{Z_k}  $\leftarrow$  {X_k}
For each individual Z_k (k= 1,2,3.....N)
For each independent variable indether to immigration to Z_k
If immigrating then  $s \in [1,n]$ 
Use  $\lambda_k$  to probabilistically decide wh
Use { $\mu_i$ } to probabilistically select the emigrating individual X_j
Z_k(s)  $\leftarrow$  X_j(s)
End if
Next independent variable index : s  $\leftarrow$  s+1
Probabilistically mutate Z_k
Next individual : K  $\leftarrow$  K+1
{X_k}  $\leftarrow$  {Z_k}
Next generation
    
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Fig. 2 Mutation

## III. CONVERGENCE OF BBO ALGORITHM

Different optimization algorithms are local search algorithm out of which meta-heuristic algorithms are appropriate for obtaining global optimization. Many algorithms may be trapped in a local optimum. As we have shown the BBO algorithm has good global convergence property. It can be more significant for global optimization. In order to depict the BBO algorithm has good convergence for various function.

TABLE I THE BENCHMARK TEST FUNCTIONS

Name	Test Function	Nature of Function	Search Space	F <sub>min</sub>
Ackley	$f_1(x) = -20e^{-0.02\sqrt{D^{-1}\sum_{i=1}^D x_i^2} - D^{-1}\sum_{i=1}^D \cos(2\pi x_i) + 20 + e}$	Continuous, Differentiable, Non-Separable, Scalable, Multi-Model	[-32,32]	0
Beale	$f_2(x) = (1.5 - x_1 + x_1 x_2)^2 + (2.25 - x_1 + x_1 x_2^2)^2 + (2.625 - x_1 + x_1 x_2^3)^2$	Continuous, Differentiable, Non-Separable, Non-Scalable, Multi-Model	[-4.5,4.5]	0
Booth	$f_3(x) = (x_1 + 2x_2 - 7)^2 + (2x_1 + x_2 - 5)^2$	Continuous, Differentiable, Non-Separable, Non-Scalable, Uni-Model	[-10,10]	0
Carrom table	$f_4(x) = -\left\{ \cos(x_1)\cos(x_2)e^{\left 1 - \frac{(x_1^2 + x_2^2)^{0.5}}{\pi}\right }\right\}^2 / 30$	Continuous, Differentiable, Separable, Scalable, Multi-Model	[-10,10]	24.1566
Crowned cross	$f_5(x) = 0.0001 \left[ \sin(x_1)\sin(x_2)e^{\left 100 - \frac{(x_1^2 + x_2^2)^{0.5}}{\pi}\right } \right] / 1$	Continuous, Differentiable, Separable, Scalable, Multi-Model	[-10,10]	0.0001
Corss in Tray	$f_6(x) = -0.0001 \left[ \left  \sin(x_1)\sin(x_2)e^{\left 100 - \frac{(x_1^2 + x_2^2)^{0.5}}{\pi}\right } \right  \right] + 1$	Continuous, Non-Separable, Non-Scalable, Multi-Model	[-10,10]	-2.0626
Easom	$f_7(x) = -\cos(x_1)\cos(x_2)\exp[-(x_1 - \pi)2 - (x_2 - \pi)2]$	Continuous, Differentiable, Separable, Non-Scalable, Multi-Model	[-100,100]	-1
Powell	$f_8(x) = \sum_{i=1}^n  x_i ^{i+1}$	Continuous, Non-Differentiable, Separable, Uni-Model, Convex dimensional	[-1,1]	0
Pen Holder	$f_9(x) = -\text{Exp} \left[ \left  \cos(x_1)\cos(x_2)e^{\left 1 - \frac{(x_1^2 + x_2^2)^{0.5}}{\pi}\right } \right  \right]^{-1}$	Continuous, Differentiable, Non-Separable, Non-Scalable, Multi-Model	[-11,11]	-0.96354
Rastigin	$f_{10}(x) = 10d + \sum_{i=1}^d [x_i^2 - 10 \cos(2\pi x_i)]^2$	Continuous, Differentiable, Separable, Scalable, Multi-Model	[-5.12,5.12]	0
Schwefel	$f_{11}(x) = \sum_{i=1}^n  x_i $	Continuous, Non-Differentiable, Separable, Scalable, Uni-Model	[-10,10]	0
Sphere	$f_{12}(x) = \sum_{i=1}^n x_i^2$	Continuous, Differentiable, Separable, Scalable, Multi-Model	[-100,100]	0

TABLE II THE BENCHMARK TEST FUNCTIONS

Functions Name	Population Size	Number of Iteration	Dim.	Best	Global Minima
Ackley	200	1000	2	0.000000	0
	100	700		0.00000695	
	50	500		0.0007155	
Beale	200	1000	2	0.00000001	0
	100	700		0.00000004	
	50	500		0.00000288	
Booth	200	1000	2	.0001194	0
	100	700		0.000347	
	50	500		0.00001051	
CarronTable	200	1000	2	-24.1568	-24.1568
	100	700		-24.1568	
	50	500		-24.1568	
Crowned Cross	200	1000	2	0.00012072	0.00014
	100	700		0.00011774	
	50	500		0.00015515	
Crossin Tray	200	1000	2	-2.0626	-2.0626
	100	700		-2.0626	
	50	500		-2.0626	
Easom	200	1000	2	-1	-1
	100	700		-1	
	50	500		-1	
Penholder	200	1000	2	-0.96353	-0.96353
	100	700		-0.96353	
	50	500		-0.96353	
Powell	200	1000	2	Nan	0
	100	700		Nan	
	50	500		Nan	
Rastrigin	200	1000	2	0	0
	100	700		0	
	50	500		0.0000355	
Schweffel	200	1000	2	-837.9658	0
	100	700		-837.9658	
	50	500		-837.9658	
Sphere	200	1000	2	2.6322x10-12	0
	100	700		4.6313x10-10	
	50	500		5.5429x10-8	

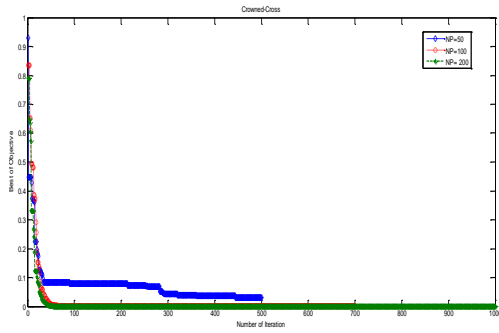


Fig. 3 Ackley

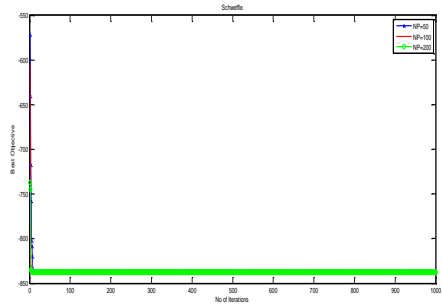


Fig. 4 Beale

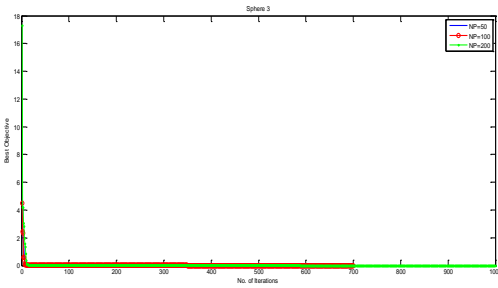


Fig. 5 Booth

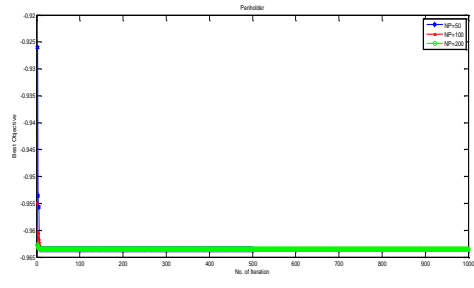


Fig. 9 Easom

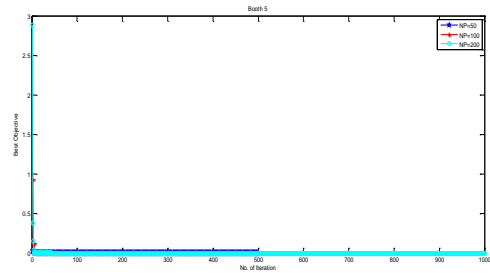


Fig. 6 Carrom Table

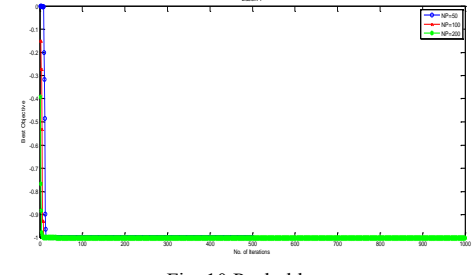


Fig. 10 Penholder

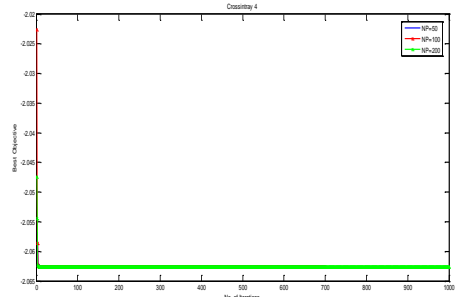


Fig. 7 Crowned Cross

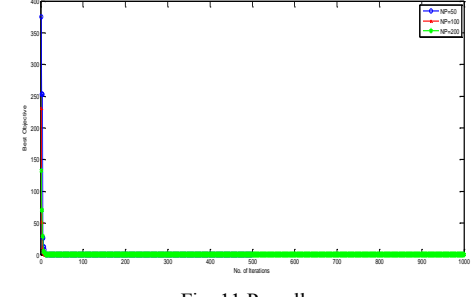


Fig. 11 Powell

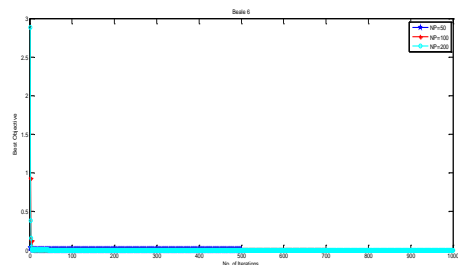


Fig. 8 Crossin Tray

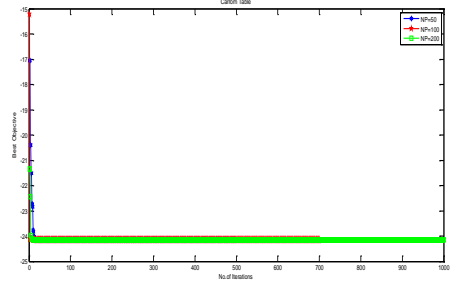


Fig. 12 Rastrigin

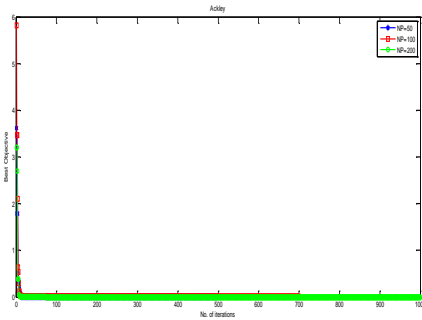


Fig. 13 Schwefel

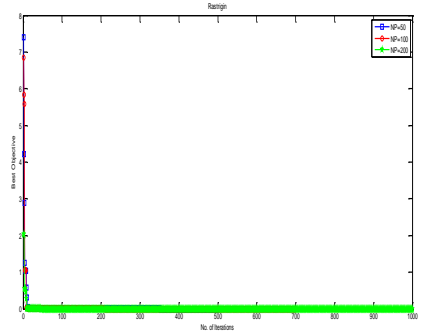


Fig. 14 Sphere

When we choose the following population size (NP) are 50,100,200 corresponding to the No. of Iterations are 100, 200, 300 respectively with the consideration of 12 Benchmark functions are performing divergence in the given range of interval. However we change the slight variation in population size (NP) are 50,100,200 corresponding to the No. of Iterations are 500, 700, 1000 respectively which performing on above said benchmark function then it provides very smoothly convergence.

## VI. CONCLUSION

In this investigation, The convergence rate of the various function such as Ackley, Beale, Booth, Carron-table, Crowned-cross, Cross-in-tray, Easom, Penholder, Powell, Rastrigin, Schwefel, Sphere have been examined by BBO algorithm, it is observed that the all considered fitness function are converged suitably. Obtained computational results are very much near to the defined results of the problem.

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