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An Efficient Reverse Converter for the Four Non Coprime Moduli Set $\{2n, 2n-1, 2n-2, 2n-3\}$

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Abstract **- In this paper, residue to binary conversion is presented for the four moduli set $\{2n, 2n-1, 2n-2, 2n-3\}$ sharing a common factor. A new and efficient converter for the moduli set using multipliers, carry saves and modular adders is proposed based on a cyclic jump approach. A theoretical hardware implementation and comparison with a state-of- the- art scheme showed that the proposed scheme performed better. The 4- moduli set selected provides a larger dynamic range which is needed for Digital Signal Processing (DSP) applications [7].**
Keywords: **Residue Number System, Non-Coprime Moduli Set, Dynamic Range, Cyclic Jump Technique**

I. INTRODUCTION

Residue Number System (RNS) is an emerging area of research. This is because of its suitability for the implementation of high-speed digital signal processing devices and its inherent parallelism, modularity, fault tolerance and carry free propagation properties [9].

Arithmetic operations such as addition and multiplication are performed more easily and efficiently in RNS than conventional two's complement number systems [8].The traditional moduli set $\{2^n+1,\ 2^n,\ 2^n-1\}$, has been one of the most popularly studied in RNS. The moduli set $\{2n, 2n-1, 2n-2, 2n-3\}$which shares a common factor of 2 between the first and third moduli has been used.

This moduli set is of study significance since it offers consecutiveness and allows for equal width adders and multipliers in hardware design which the traditional moduli sets do not provide [7].

II. FUNDAMENTALS OF RESIDUE NUMBER SYSTEM

RNS is defined in terms of a set of relatively prime moduli set $\{m_i\}_{i=1,k}$ suchthat, the greatest common divisor(gcd) of $(m_i, m_j) = 1$ for $i \neq j$, while $M = \prod_{i=1}^{k} m_i$, is the dynamic range. The residues of a decimal number X can be obtained as $x_i = |X|_{m_i}$, thus X can be represented in RNS as $X = (x_1, x_2, x_3, \dots, x_k)$, $0 \leq x_i \leq m_i$. This representation is unique for any integer $X \in [0, M-1]$. $|X|_{m_i}$is the modulo operation of X with respect to m_i[1],[5].

A. Chinese Remainder Theorem

The Chinese Remainder Theorem (CRT) can be used to backward convert the residue digits $(x_1, x_2, \dots, x_n)$ of the moduli set $\{m_1, m_2, \dots, m_n\}$ to its decimal number (X) as follows; For a moduli set $\{m_i\}_{i=1,N}$ with the dynamic range $M = \prod_{i=1}^{k} m_i$, the residue number $(x_1, x_2, x_3, \dots, x_N)$ can be converted into the decimal number X, according to the CRT as ;

$$X = \left| \sum_{i=1}^{N} \ell_i \, |k_i x_i|_{m_i} \right|_M \qquad (1)$$

Where,

$$M = \prod_{i=1}^{N} m_i \ ;$$

$$\ell_i = \frac{M}{m_i}\ ; \quad |k_i \times \ell_i|_{m_i} = 1 \text{ [4],[6]}$$

B. Mixed Radix Conversion

The Mixed Radix Conversion (MRC) approach serves as an alternative method to the CRT for performing reverse conversion. However, this method does not involve the use of the large modulo-M computation as is required by the CRT. This method is used to perform `residue to binary conversion of (x_1, x_2, x_3) based on the moduli set $\{m_1, m_2, \dots m_3\}$ as follows;

$$X = a_1 + a_2 m_1 + a_3 m_1 m_2 + a_n m_1 m_2 m_3 \dots m_{k-1} \quad (2)$$

Where $a_{i,i=1,k}$ are the Mixed Radix Digits (MRDs) which can be computed below as shown in [2],[3],[10];

$$a_1 = x_1$$

$$a_2 = \left| (x_2 - a_1) |m_1^{-1}|_{m_2} \right|_{m_2}$$

$$a_3 = \left| ((x_3 - a_1) |m_1^{-1}|_{m_3} - a_2) |m_2^{-1}|_{m_3} \right|_{m_3}$$

$$\vdots$$

$$a_k = \left| (((x_k - a_1) |m_1^{-1}|_{m_k} - a_2) |m_2^{-1}|_{m_k} - \dots - a_{k-1}) |m_{k-1}^{-1}|_{m_k} \right|_{m_k}$$

III. THE PROPOSED CONVERSION TECHNIQUE

A cyclic jump method is presented in this paper. The technique employs an initial position being equal to the first residue and then jumps to new locations until a final point is reached. The various jumps are then summed when all residues turn to zero, to arrive at the decimal number X. This technique is an MRC based approach.

A. Jump Technique for the New 4- Moduli Set $\{2n, 2n-1, 2n-2, 2n-3\}$

1. Odd Case n Values

Given the general Moduli set $\{2n, 2n-1, 2n-2, 2n-3\}$ sharing a common factor 2 between the first and third moduli, then for $n \geq 3$ being odd, the conversion process is as shown;

For the particular case of $n = 3$
Then $\{2n, 2n-1, 2n-2, 2n-3\}$ generates $\{6,5,4,3\}$
Given the moduli set $\{m_1, m_2, m_3, m_4\} = \{6,5,4,3\}$ and a decimal number $X = 45$ within the acceptable range, the reverse conversion is as follows;

$\{m_1, m_2, m_3, m_4\} = \{6,5,4,3\}$ will produce the residue set $\{r_1, r_2, r_3, r_4\} = \{3,0,1,0\}$

Applying the conversion technique, we proceed as follows;

Step 1: First jump (J_1) is equal to r_1

i.e $J_1 = r_1 = 3$

First location after jump$(J_1)$is given as $L_1 = X - J_1$
$= 45 - 3 = 42$

$$\text{Also } L_1 = r - J_1 = \begin{bmatrix} |r_1 - J_1|m_1 = r_1' \\ |r_2 - J_1|m_2 = r_2' \\ |r_3 - J_1|m_3 = r_3' \\ |r_4 - J_1|m_4 = r_4' \end{bmatrix}$$

$$= \begin{bmatrix} |3-3|_6 = |0|_6 = 0 \\ |0-3|_5 = |-3|_5 = 2 \\ |1-3|_4 = |-2|_4 = 2 \\ |0-3|_3 = |-3|_3 = 0 \end{bmatrix} \text{ where } (r_1', r_2', r_3', r_4') = (0,2,2,0)$$

Step 2: Second jump (J_2) is defined by:

$J_2 = m_1 k_2$ and $|r_2' - J_2|m_2 = 0$
$|r_2' - m_1 k_2|m_2 = 0$

$k_2 = \left|\frac{r_2'}{m_1}\right| m_2 = \left|\frac{2}{6}\right|_5 = \left|\frac{1}{3}\right|_5 = \left|\frac{6}{3}\right|_5 = |2|_5 = 2$
$J_2 = m_1 k_2 = 6.2 = 12$
The second location(L_2) after the jump (J_2) is given by;

$$L_2 = r - J_1 - J_2 = L_1 - J_2 = \begin{bmatrix} |r_1 - J_2|m_1 = r_1'' \\ |r_2 - J_2|m_2 = r_2'' \\ |r_3 - J_2|m_3 = r_3'' \\ |r_4 - J_2|m_4 = r_4'' \end{bmatrix}$$

$L_2 = L_1 - J_2 = 42 - 12 = 30$

$$= \begin{bmatrix} |3-3|_6 = |0|_6 = 0 \\ |0-3|_5 = |-3|_5 = 2 \\ |1-3|_4 = |-2|_4 = 2 \\ |0-3|_3 = |-3|_3 = 0 \end{bmatrix}$$

where $(r_1'', r_2'', r_3'', r_4'') = (0,2,2,0)$

Step 3: Third jump(J_3) is given by:

$J_3 = m_1 m_2\, k_3$ and $|r_3'' - J_3|m_3 = 0$

$$= |r_3'' - m_1 m_2 k_3|_{m_3} = 0\,, K_3 = \left|\frac{r_3''}{m_1 m_2}\right| m_3$$

$K_3 = \left|\frac{r_3''}{m_1 m_2}\right| m_3 = \left|\frac{2}{6.5}\right|_4 = \left|\frac{2}{30}\right|_4 = |1|_4 = 1$
$J_3 = m_1 m_2\, k_3 =_3 6.5.1 = 30$

Third Location (L_3) after the jump J_3 is given by;

$$L_3 = r - J_1 - J_2 - J_3 = L_2 - J_3$$
$$= \begin{bmatrix} |r_1 - J_3|m_1 = r_1''' \\ |r_2 - J_3|m_2 = r_2''' \\ |r_3 - J_3|m_3 = r_3''' \\ |r_4 - J_3|m_4 = r_4''' \end{bmatrix}$$

$L_3 = L_2 - J_3 = 30 - 30 = 0$

$$= \begin{bmatrix} |0-30|_6 = |0|_6 = 0 \\ |0-30|_5 = |-30|_5 = 0 \\ |2-30|_4 = |-28|_4 = 0 \\ |0-30|_3 = |-30|_3 = 0 \end{bmatrix}$$

where $(r_1''', r_2''', r_3''', r_4''') = (0,0,0,0)$

Step 4: Fourth jump(J_4) is given by:

$J_4 = m_1 m_2\, m_3\, k_4$ and $|r_4''' - J_4|m_4 = 0$

$$= |r_4''' - m_1 m_2 m_3 k_4|_{m_4} = 0\,, K_4 = \left|\frac{r_4'''}{m_1 m_2 m_3}\right| m_4$$

$K_4 = \left|\frac{r_3'''}{m_1 m_2 m_3}\right| m_3 = \left|\frac{0}{6.5.4}\right|_4 = |0|_4 = 0$
$J_4 = m_1 m_2\, m_3\, k_3 = 6.5.4.0 = 0$

Fourth Location (L_4) after the jump J_4 is given by:

$$\text{Also } L_4 = r - J_1 - J_2 - J_3 - J_4 = L_3 - J_4$$
$$= \begin{bmatrix} |r_1 - J_4|m_1 = r_1'''' \\ |r_2 - J_4|m_2 = r_2'''' \\ |r_3 - J_4|m_3 = r_3'''' \\ |r_4 - J_4|m_4 = r_4'''' \end{bmatrix}$$

$L_4 = L_3 - J_4 = 0 - 0 = 0$

$$= \begin{bmatrix} |0-0|_6 = |0|_6 = 0 \\ |0-0|_5 = |0|_5 = 0 \\ |0-0|_4 = |0|_4 = 0 \\ |0-0|_3 = |0|_3 = 0 \end{bmatrix}$$

where $(r_1^{''''}, r_2^{''''}, r_3^{''''}, r_4^{''''}) = (0,0,0,0)$

The decimal number is computed as follows:

$$X = J_1 + J_2 + J_3 + J_4$$

$X = 3 + 12 + 30 + 0 = 45$

The algorithm generates the decimal number used for the residue for all numbers in a very fast manner for the moduli set $\{2n, 2n-1, 2n-2, 2n-3\}$, given any odd n value.

2. Even Case n Values

Given the selected 4-Moduli set $\{2n, 2n-1, 2n-2, 2n-3\}$ sharing a common factor 2 between the first and third moduli, then for $n \geq 3$ being even, the conversion process is as follows;

For the particular case of $n = 10$, then $\{2n, 2n-1, 2n-2, 2n-3\}$ yields $\{20,19,18,17\}$.

Given the moduli set $\{m_1, m_2, m_3, m_4\} = \{20,19,18,17\}$ and a decimal number $X = 100$ within the acceptable range, the reverse conversion proceeds as follows;

$\{m_1, m_2, m_3, m_4\} = \{20,19,18,17\}$ produces the residue set $\{r_1, r_2, r_3, r_4\} = \{0,5,10,15\}$

Applying the conversion technique, gives;

Step 1: First jump (J_1) is equal to r_1.

i.e $J_1 = r_1 = 0$

First location after jump$(J_1)$is given as;

$$L_1 = X - J_1 = 100 - 0 = 100$$

Also $L_1 = r - J_1 = \begin{bmatrix} |r_1 - J_1|m_1 = r_1^{'} \\ |r_2 - J_1|m_2 = r_2^{'} \\ |r_3 - J_1|m_3 = r_3^{'} \\ |r_4 - J_1|m_4 = r_4^{'} \end{bmatrix}$

$= \begin{bmatrix} |0-0|_{20} = |0|_{20} = 0 \\ |5-0|_{19} = |5|_{19} = 5 \\ |10-0|_{18} = |10|_{18} = 10 \\ |15-0|_{17} = |15|_{17} = 15 \end{bmatrix}$ where $(r_1^{'}, r_2^{'}, r_3^{'}, r_4^{'}) =$ (0,5,10,15)

Step 2: Second jump (J_2) is defined by:

$J_2 = m_1 k_2$ and $|r_2^{'} - J_2|m_2 = 0$ $|r_2^{'} - m_1 k_2|m_2 = 0$

$k_2 = \left|\frac{r_2^{'}}{m_1}\right| m_2 = \left|\frac{5}{20}\right|_{19} = \left|\frac{100}{20}\right|_{19} = |5|_{19} = 5$

$J_2 = m_1 k_2 = 20.5 = 100$

The second location(L_2) after the jump (J_2) is given by:

$$L_2 = r - J_1 - J_2 = L_1 - J_2$$

$L_2 = L_1 - J_2 = 100 - 100 = 0$

Also $L_2 = r - J_1 - J_2 = L_1 - J_2$

$$= \begin{bmatrix} |r_1 - J_2|m_1 = r_1^{''} \\ |r_2 - J_2|m_2 = r_2^{''} \\ |r_3 - J_2|m_3 = r_3^{''} \\ |r_4 - J_2|m_4 = r_4^{''} \end{bmatrix}$$

$= . \begin{bmatrix} |0-100|_{20} = |-100|_{20} = 0 \\ |5-100|_{19} = |-95|_{19} = 0 \\ |10-100|_{18} = |-90|_{18} = 0 \\ |15-100|_{17} = |-85|_{17} = 0 \end{bmatrix}$

where $(r_1^{''}, r_2^{''}, r_3^{''}, r_4^{''}) = (0,0,0,0)$

Step 3: Third jump(J_3) is given by:

$J_3 = m_1 m_2\, k_3$ and $|r_3^{''} - J_3|m_3 = 0$

$$= \left|r_3^{''} - m_1 m_2 k_3\right|_{m_3} = 0\, , K_3 = \left|\frac{r_3^{''}}{m_1 m_2}\right|_{m_3}$$

$K_3 = \left|\frac{r_3^{''}}{m_1 m_2}\right|_{m_3} == \left|\frac{0}{20.19}\right|_{18} = |0|_4 = 0$

$J_3 = m_1 m_2\, k_3 = 20.19.0 = 0$

Third Location (L_3) after the jump J_3 is given by:

$$L_3 = r - J_1 - J_2 - J_3 = L_2 - J_3$$

$L_3 = L_2 - J_3 = 0 - 0 = 0$

Also $L_3 = r - J_1 - J_2 - J_3 = L_2 - J_3$

$$= \begin{bmatrix} |r_1 - J_3|m_1 = r_1^{'''} \\ |r_2 - J_3|m_2 = r_2^{'''} \\ |r_3 - J_3|m_3 = r_3^{'''} \\ |r_4 - J_3|m_4 = r_4^{'''} \end{bmatrix}$$

$$= \begin{bmatrix} |0-0|_{20} = |0|_{20} = 0 \\ |0-0|_{19} = |0|_{19} = 0 \\ |0-0|_{18} = |0|_{18} = 0 \\ |0-0|_{17} = |0|_{17} = 0 \end{bmatrix}$$

where $(r_1^{'''}, r_2^{'''}, r_3^{'''}, r_4^{'''}) = (0,0,0,0)$

Step 4: Fourth jump(J_4) is given by;

$J_4 = m_1 m_2\, m_3\, k_4$ and $\left|r_4^{'''} - J_4\right|_{m_4} = 0$

$$= \left|r_4^{'''} - m_1 m_2 m_3 k_4\right|_{m_4} = 0\ , K_4 = \left|\frac{r_4^{'''}}{m_1 m_2 m_3}\right|_{m_4}$$

$K_4 = \left|\frac{r_4^{'''}}{m_1 m_2 m_3}\right|_{m_4} = \left|\frac{0}{20.19.18}\right|_4 = |0|_4=0$

$J_4 = m_1 m_2\, m_3\, k_4 = 20.19.18.0 = 0$

Fourth Location (L_4) after the jump J_4 is given by:

$$L_4 = r - J_1 - J_2 - J_3 - J_4 = L_3 - J_4$$

$L_4 = L_3 - J_4 = 0 - 0 = 0$

Also,

$$L_4 = r - J_1 - J_2 - J_3 - J_4 = L_3 - J_4 = \begin{bmatrix} |r_1 - J_4|m_1 = r_1^{''''} \\ |r_2 - J_4|m_2 = r_2^{''''} \\ |r_3 - J_4|m_3 = r_3^{''''} \\ |r_4 - J_4|m_4 = r_4^{''''} \end{bmatrix}$$

$$= \begin{bmatrix} |0-0|_{20} = |0|_{20} = 0 \\ |0-0|_{19} = |0|_{19} = 0 \\ |0-0|_{18} = |0|_{18} = 0 \\ |0-0|_{17} = |0|_{17} = 0 \end{bmatrix}$$

where $(r_1^{''''}, r_2^{''''}, r_3^{''''}, r_4^{''''}) = (0,0,0,0)$

The decimal number is computed as follows:

$$X = J_1 + J_2 + J_3 + J_4$$

$$X = 0 + 100 + 0 + 0 = 100$$

Since the algorithm is able to get back the decimal number which we used for the residue set for this case and several others, then the conversion algorithm works well for the conversion of the moduli set,$\{2n, 2n-1, 2n-2, 2n-3\}$, given any even n value.

IV. BINARY REPRESENTATION FOR THE MODULI SET$\{2n, 2n-1, 2n-2, 2n-3\}$

Given the moduli set $\{2n, 2n-1, 2n-2, 2n-3\}$, its binary representation is given as follows;

Step 1: First jump (J_1), which is equal to first residue r_1. i.e $J_1 = r_1$

First location L_1 is defind as $L_1 = r - J_1$

$r_1 = (r_{1,2n} \dots r_{1,0})$

$r_2 = (r_{2,2n-3} \dots r_{2,1}, r_{2,0})$

$r_3 = (r_{3,2n-2} \dots r_{3,1}, r_{3,0})$

$r_4 = (r_{4,2n-1} \dots r_{4,1}, r_{4,0})$

$$Thus\ L_1 = r - J_1 = \begin{bmatrix} |r_1 - J_1|m_1 = r_1^{'} \\ |r_2 - J_1|m_2 = r_2^{'} \\ |r_3 - J_1|m_3 = r_3^{'} \\ |r_4 - J_1|m_4 = r_4^{'} \end{bmatrix}$$

$$= \begin{bmatrix} \left|(r_{1,4} \dots r_{1,0}) - (r_{1,4} \dots r_{1,0})\right|_{2n} \\ \left|(r_{2,2n-2} \dots r_{2,1}, r_{2,0}) - (r_{1,4} \dots r_{1,0})\right|_{2n-1} \\ \left|(r_{3,2n-1} \dots r_{3,1}, r_{3,0}) - (r_{1,4} \dots r_{1,0})\right|_{2n-2} \\ \left|(r_{4,2n-1} \dots r_{4,1}, r_{4,0}) - (r_{1,4} \dots r_{1,0})\right|_{2n-3} \end{bmatrix}$$

$$= \begin{bmatrix} 0000 \\ \left|(r_{2,2n-2} \dots r_{2,1}, r_{2,0}) + (\overline{r_{1,4}} \dots \overline{r_{1,0}})\right|_{2n-1} \\ \left|(r_{3,2n-1} \dots r_{3,1}, r_{3,0}) + (\overline{r_{1,4}} \dots \overline{r_{1,0}})\right|_{2n-2} \\ \left|(r_{4,2n-1} \dots r_{4,1}, r_{4,0}) + (\overline{r_{1,4}} \dots \overline{r_{1,0}})\right|_{2n-3} \end{bmatrix}$$

$$= \begin{bmatrix} 0000 \\ r_{2,2n-3}^{'}, r_{2,0}^{'} \\ r_{3,2n-2}^{'}, r_{3,0}^{'} \\ r_{4,2n-1}^{'}, r_{4,0}^{'} \end{bmatrix}$$

Step 2: Second jump (J_2) is defined by:

$J_2 = m_1 k_2$ and$|r_2^{'} - J_2|_{m_2} = 0$

$$\ni |r_2^{'} - m_1 k_2|_{m_2} = 0 = k_2 = \left|\frac{r_2^{'}}{m_1}\right|_{m_2}$$

It is possible to continuously iterate the expression (*) by repeatedly adding the modulus m_2 until such a time that $|r_2^{'} - m_1 k_2|_{m_2} = 0$

$J_2 = m_1 k_2 = 2^2 k_2 = k_{2,2n-2}, \dots\ k_{2,0}.00$

$= J_{2,2n}, \dots J_{2,0}$

Location L_2 is given as: $L_2 = r - J_1 - J_2 = L_1 – J_2$

$$L_2 = r - J_1 - J_2 = L_1 – J_2 = \begin{bmatrix} |r_1 - J_2|m_1 = r_1^{''} \\ |r_2 - J_2|m_2 = r_2^{''} \\ |r_3 - J_2|m_3 = r_3^{''} \\ |r_4 - J_2|m_4 = r_4^{''} \end{bmatrix}$$

$$= \begin{bmatrix} 0000 \\ \left|(r_{2,2n-3} \dots r_{2,1}, r_{2,0}) - (J_{2,2n}, \dots J_{2,0})\right|_{2n-1} \\ \left|(r_{3,2n-2} \dots r_{3,1}, r_{3,0}) - (J_{2,2n}, \dots J_{2,0})\right|_{2n-2} \\ \left|(r_{4,2n-1} \dots r_{4,1}, r_{4,0}) - (J_{2,2n}, \dots J_{2,0})\right|_{2n-3} \end{bmatrix}$$

$$= \begin{bmatrix} 0000 \\ 00 \dots 00 \\ \left|(r_{3,2n-1} \dots r_{3,1}, r_{3,0}) - (J_{2,2n}, \dots J_{2,0})\right|_{2n-2} \\ \left|(r_{4,2n-1} \dots r_{4,1}, r_{4,0}) - (J_{2,2n}, \dots J_{2,0})\right|_{2n-3} \end{bmatrix}$$

$$= \begin{bmatrix} 0000 \\ 00 \ldots 00 \\ r''_{3,2n-2}, r''_{3,0} \\ r''_{4,2n-1}, r''_{3,0} \end{bmatrix}$$

Step 3: Third jump is defined by J_3 and this is given by;

$J_3 = m_1 m_2 k_3$ and $|r''_3 - J_3|m_3 = 0 \ni |r''_3 - m_1 m_2 k_3|m_3 = 0$

$$K_3 = \left|\frac{r''_3}{m_1 m_2}\right| m_3$$

$J_3 = m_1 m_2 k_3 = 2^2 2n - 1. K_3$

$= (2n-1). K_{3,2n-1}, \ldots\ K_{3,0} 00$
$= 2n - 1. K_{3,2n+1}, \ldots\ K_{3,0}$
$= J_{3,4n}, \ldots J_{3,0}$

Third Location (L_3) after the jump J_3 is given by:

$$r_1 – J_1 – J_2 – J_3 = L_2 – J_3 = \begin{bmatrix} |r_1 - J_3|m_1 = r'''_1 \\ |r_2 - J_3|m_2 = r'''_2 \\ |r_3 - J_3|m_3 = r'''_3 \\ |r_4 - J_3|m_4 = r'''_4 \end{bmatrix}$$

$$= \begin{bmatrix} 0000 \\ \left|(r_{2,2n-3} \ldots r_{2,1}, r_{2,0}) - (J_{2,2n}, \ldots J_{2,0})\right|_{2n-1} \\ \left|(r_{3,2n-2} \ldots r_{3,1}, r_{3,0}) - (J_{2,2n}, \ldots J_{2,0})\right|_{2n-2} \\ \left|(r_{4,2n-1} \ldots r_{4,1}, r_{4,0}) - (J_{2,2n}, \ldots J_{2,0})\right|_{2n-3} \end{bmatrix}$$

$$= \begin{bmatrix} 0000 \\ 00 \ldots 00 \\ 00 \ldots 00 \\ \left|(r_{4,2n-1} \ldots r_{4,1}, r_{4,0}) - (J_{2,2n}, \ldots J_{2,0})\right|_{2n-3} \end{bmatrix}$$

$$= \begin{bmatrix} 0000 \\ 00 \ldots 00 \\ 00 \ldots 00 \\ r''_{4,2n-1}, r''_{3,0} \end{bmatrix}$$

Step 4: Fourth jump(J_4) is given by:

$J_4 = m_1 m_2\ m_3\ k_4$ and $\left|r'''_4 - J_4\right|_{m_4} = 0$

$$= \left|r'''_4 - m_1 m_2 m_3 k_4\right|_{m_4} = 0\ , K_4 = \left|\frac{r'''_4}{m_1 m_2 m_3}\right|_{m_4}$$

$K_4 = \left|\frac{r'''_4}{m_1 m_2 m_3}\right|_{m_4} = \left|\frac{0}{20.19.18}\right|_4 = |0|_4 = 0$

$J_4 = m_1 m_2\ m_3\ k_4 = 20.19.18.0 = 0$

Fourth Location (L_4) after the jump J_4 is given by:

$$L_4 = r - J_1 - J_2 - J_3 - J_4 = L_3 - J_4$$

Also, $L_4 = r - J_1 - J_2 - J_3 - J_4 = L_3 - J_4$

$$= \begin{bmatrix} |r_1 - J_4|m_1 = r''''_1 \\ |r_2 - J_4|m_2 = r''''_2 \\ |r_3 - J_4|m_3 = r''''_3 \\ |r_4 - J_4|m_4 = r''''_4 \end{bmatrix}$$

$$= \begin{bmatrix} 0000 \\ \left|(r_{2,2n-3} \ldots r_{2,1}, r_{2,0}) - (J_{2,2n}, \ldots J_{2,0})\right|_{2n-1} \\ \left|(r_{3,2n-2} \ldots r_{3,1}, r_{3,0}) - (J_{2,2n}, \ldots J_{2,0})\right|_{2n-2} \\ \left|(r_{4,2n-1} \ldots r_{4,1}, r_{4,0}) - (J_{2,2n}, \ldots J_{2,0})\right|_{2n-3} \end{bmatrix}$$

$$= \begin{bmatrix} 0000 \\ 00 \ldots 00 \\ 00 \ldots 00 \\ \left|(r_{4,2n-1} \ldots r_{4,1}, r_{4,0}) - (J_{2,2n}, \ldots J_{2,0})\right|_{2n-3} \end{bmatrix}$$

$$= \begin{bmatrix} 0000 \\ 00 \ldots 00 \\ 00 \ldots 00 \\ r''_{4,2n-1}, r''_{3,0} \end{bmatrix}$$

where $(r''''_1, r''''_2, r''''_3, r''''_4) = (0,0,0,0)$

The binary number is computed as follows;

$X = r_{1,3} \ldots r_{1,0} + J_{2,2n}, \ldots J_{2,0} + J_{3,4n}, \ldots J_{3,0} + J_{4,2n,\ldots,}J_{4,0}$

$$= \underbrace{\underbrace{00 \ldots 00}_{(4n-3)bit}\overbrace{r_{1,3} \ldots r_{1,0}}^{4\ bit} + \underbrace{00 \ldots 00}_{2n\ bit}\overbrace{J_{2,2n}, \ldots J_{2,0}}^{(2n+1)bit} + \# \quad \underbrace{00 \ldots 00}_{2n\ bit}\underbrace{J_{3,4n}, \ldots J_{3,0}}_{(4n+1)\ bit} + \underbrace{J_{4,2n-1}, \ldots J_{4,0}}_{(4n+1)\ bit}}_{(4n+1)bit}$$

V. HARDWARE IMPLEMENTATION OF THE PROPOSED SCHEME

The hardware implementation for the new 4- moduli set reverse converter based on the cyclic Jump Technique is shown in figure 1.

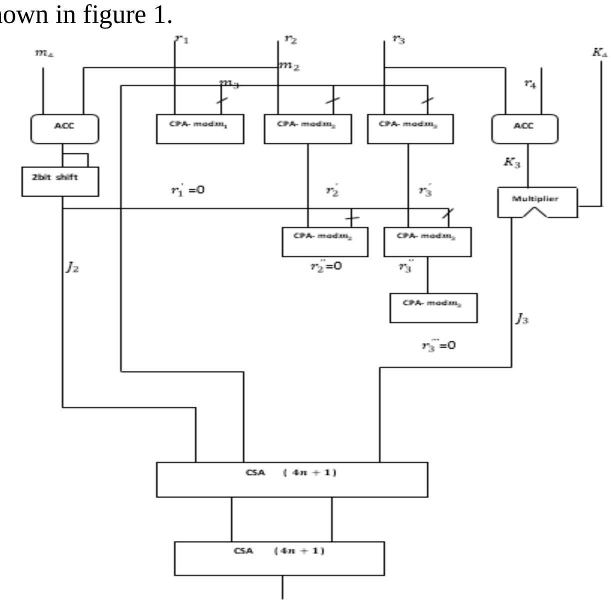

Fig. 1 Architecture for the proposed cyclic jump method

VI. CONCLUSION

The paper proposed a new converter for the special 4-moduli set $\{2n, 2n-1, 2n-2, 2n-3\}$sharing a common factor of 2. The proposed converter is very fast in performing reverse conversions compared with the converter presented in [9], as it generates the decimal value in only four steps. The converter is best suited for DSP applications requiring very large dynamic ranges.

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